Mid-Term Exam: Algebra (8– 4 –2013) Prep-Year Model Answer

[1](a)A square matrix A is called symmetric if $\mathbf{A^t = A}$.

(b)A square matrix A has inverse A^{-1} if $|\mathbf{A}| \neq \mathbf{0}$.

(c) A linear system AX = B is called consistent if **it has a solution**.

(d)The rank of a matrix A is **the number of non zero rows after applying elementary operations to reduce it as upper triangular matrix.**

---(4 Marks)

[2]The augmented matrix of the linear system: x – y + 2z = 3, – x + y – 2z = 2, 2x + y + z = 1

is: $G = \left[\begin{array}{ccc:c} 1 & -1 & 2 & 3 \\ -1 & 1 & -2 & 2 \\ 2 & 1 & 1 & 1 \end{array}\right]$. Apply the operations, $R_1 + R_2$, $-2R_1 + R_3$ and then $R_2 \leftrightarrow R_3$

$$G \sim \left[\begin{array}{ccc:c} 1 & -1 & 2 & 3 \\ 0 & 0 & 0 & 5 \\ 0 & 3 & -3 & -5 \end{array}\right] \sim \left[\begin{array}{ccc:c} 1 & -1 & 2 & 3 \\ 0 & 3 & -3 & -5 \\ 0 & 0 & 0 & 5 \end{array}\right]$$

We see that, rank A = 2 and rank G = 3. Then, this linear system has no solution.

---(5 Marks)

[3] If $A = \begin{bmatrix} 1 & 2 & 3 \\ 0 & 2 & 1 \end{bmatrix}$, $B = \begin{bmatrix} 2 & 1 \\ 0 & 2 \\ 3 & 1 \end{bmatrix}$. Then A + B does not exist, |A| does not exist.

$$A + B^t = \begin{bmatrix} 1 & 2 & 3 \\ 0 & 2 & 1 \end{bmatrix} + \begin{bmatrix} 2 & 0 & 3 \\ 1 & 2 & 1 \end{bmatrix} = \begin{bmatrix} 3 & 2 & 6 \\ 1 & 4 & 2 \end{bmatrix}$$

$$BA = \begin{bmatrix} 2 & 1 \\ 0 & 2 \\ 3 & 1 \end{bmatrix} \begin{bmatrix} 1 & 2 & 3 \\ 0 & 2 & 1 \end{bmatrix} = \begin{bmatrix} 2 & 6 & 7 \\ 0 & 4 & 2 \\ 3 & 8 & 10 \end{bmatrix}$$

$|BA| = 2(40 - 16) - 6(0 - 6) + 7(0 - 12) = 0$

---*(5 Marks)*

[4]If $A = \begin{bmatrix} 2 & 2 \\ 2 & -1 \end{bmatrix}$. Then $|A - \lambda I| = \begin{vmatrix} 2-\lambda & 2 \\ 2 & -1-\lambda \end{vmatrix} = (2-\lambda)(-1-\lambda) - 4 = 0$

Then, $\lambda^2 - \lambda - 6 = 0$. Then, the eigenvalues: $\lambda_1 = 3$, $\lambda_2 = -2$.

From the equation, $\begin{bmatrix} 2-\lambda & 2 \\ 2 & -1-\lambda \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = 0$

For : $\lambda_1 = 3$, $\begin{bmatrix} -1 & 2 \\ 2 & -4 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = 0$

Then – x + 2y = 0, 2x – 4y = 0

Then x = 2y = any number except 0

Put y = 1, we get x = 2 and the corresponding eigenvector is: $X_1 = \begin{bmatrix} 2 \\ 1 \end{bmatrix}$

For : $\lambda_2 = -2$, $\begin{bmatrix} 4 & 2 \\ 2 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = 0$

Then 4x + 2y = 0, 2x + y = 0

Then y = – 2x = any number except 0

Put x = 1, we get y = – 2 and the corresponding eigenvector is: $X_2 = \begin{bmatrix} 1 \\ -2 \end{bmatrix}$

---*(6 Marks)*